The more challenging problems are marked with $*$. Be concise when describing a Turing machine. It is like writing pseudocodes. It suffices to present the most important ideas behind your Turing machines. You do not need to give all the details, e.g., the set of states and the transition function. Check the course website for more info about homeworks. CC: Computational Complexity. TC: Introduction to the Theory of Computation. MA: Computational Complexity: A Modern Approach. LC: Lectures in Computational Complexity (at `http://pages.cs.wisc.edu/~jyc/book.pdf`).

1. Exercise 17.2 on page 359 of MA: Show that the problem of computing the permanent for matrices with integer entries (note that the entries can be negative in general) can be solved by a deterministic polynomial-time algorithm with access to a #SAT oracle.

2. Let H denote a universal family of hash functions from $U = \{0,1\}^n$ to $T = \{0,1\}^m$, where $n > m$. Let $S \subseteq U$ be a set of size at least 2^m. Show that if h is drawn uniformly at random from H, then

$$\Pr_{h \in H}\big[h(S) \neq T\big] \leq \frac{2^{2m+1}}{|S|}.$$

Note that by $h(S) \neq T$, we mean that there exists a $t \in T$ such that $h(s) \neq t$ for all $s \in S$.

3. Use the idea and analysis behind Problem 2 to give an alternative proof for the result of Section 5.8 of LC: Given access to a SAT oracle, there is a randomized algorithm A that, given a SAT instance ϕ, outputs an integer $A(x)$ such that $\#\phi \leq A(x) \leq 2 \cdot \#\phi$ with high probability.

4. Exercise 13.13 on page 284 of MA: Given a fixed undirected graph G with n vertices, consider the following communication problem denoted by CIS_G. Alice receives a clique C in G, and Bob receives an independent set I. They have to communicate in order to determine $|C \cap I|$. (Note that this number is either 0 or 1.) Prove that $CC(\mathsf{CIS}_G) = O(\log^2 n)$. (Whether this $O(\log^2 n)$ upper bound is tight remains an open problem.)

5. Prove that $CC(\mathsf{CIS}_G) = \Omega(\log n)$. Moreover, if there is a constant $c \leq 2$ such that

$$CC(\mathsf{CIS}_G) = O(\log^c n)$$

holds for all graphs G, then show that for any $f : X \times Y \to \{0,1\}$,

$$CC(f) = O\left(\left(\log C^D(f)\right)^c\right),$$

where $C^D(f)$ denotes the smalles number of monochromatic rectangules needed to cover $X \times Y$.

6. Assume that Alice and Bob can flip public coins for free (e.g., imagine that there is a trusted third party that sends them a string of random bits as long as necessary for free). How many communication bits do they need now to solve EQ_n, say with error probability at most 1/3?